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Thermal Conversion Gravity: A Parameter-Free Scalar-Tensor Theory of Black Hole Singularity Avoidance

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Abstract

We construct Thermal Conversion Gravity (TCG), a scalar-tensor extension of general relativity in which a thermal scalar field $T(r)$, derived from the Unruh effect near black hole horizons, couples to the Schwarzschild geometry via a fully covariant action. The field profile $T(r) = A/\sqrt{r - r_s}$ is shown to be a self-consistent stationary solution of the curved-space Klein–Gordon equation for an effective potential $V_{\text{eff}}(T)$ entirely determined by the geometry — no free parameters. The stationarity condition fixes the conversion threshold $T_c = A/\sqrt{r_s}$, yielding the central prediction $r_{\text{conv}} = 2r_s$ with no free parameters and no fine-tuning. The matter–field coupling constant κ is fixed by consistency with the Hawking temperature, giving $\kappa = 8\pi G^5 M^5 k_B^3/(15\hbar^2 c^{13})$. The model is *fully determined* by the black hole mass M alone. We prove that the phase transition at $r = 2r_s$ is first-order from $V'''_{\text{eff}}(T_c) \neq 0$, implying a latent heat powering the outgoing flux. All three energy conditions (NEC, WEC, SEC) are satisfied at the conversion surface; backreaction corrections scale as $(\ell_P/r_s)^2$. The correlation length of quantum fluctuations satisfies $\xi_c \sim \ell_P$ — the Planck length emerges from TCG without external input from quantum gravity. Applying the Hawking evaporation rate, TCG predicts a Planck remnant of mass $M_* = M_P/4$ as the natural endpoint of black hole evaporation. The Kerr extension yields an oblate conversion surface $r_{\text{conv}}(\theta) = r_+ + \sqrt{r_s^2 - 4a^2} + (2a^2 r_+ \sin^2\theta)/r_s^2 + \mathcal{O}(a^4)$, testable with the Event Horizon Telescope. This paper is explicitly speculative.

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I. INTRODUCTION

The Schwarzschild singularity at $r = 0$ represents a fundamental breakdown of general relativity (GR). Numerous approaches have been proposed to resolve or replace it, including loop quantum gravity [16, 17], non-commutative geometry [18], regular black hole models [13–15], and scalar-tensor modifications [19, 22]. A common feature of these approaches is the introduction of new degrees of freedom — either quantum geometric, or additional fields — whose origin is postulated rather than derived.

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We propose a different approach. The Unruh effect [8] establishes that the near-horizon geometry already contains a diverging temperature field $T_U(r) \to \infty$ as $r \to r_s$. In standard GR, this temperature is a *consequence* of the geometry — an observable effect, not a dynamical degree of freedom. We promote it to a dynamical scalar field and demand self-consistency: the field profile must satisfy its own Klein–Gordon equation in the Schwarzschild background. This single requirement — without any free parameters — fixes the effective potential, the conversion threshold, and the conversion radius.

The result is Thermal Conversion Gravity (TCG): a scalar-tensor theory in which ordinary matter undergoes a first-order phase transition at $r_{\rm conv} = 2r_s$, before reaching the singularity. The Planck length and a Planck remnant of mass $M_P/4$ emerge from the theory without quantum gravity input.

This paper is self-contained and does not require the companion Penetron conjecture [1], which addresses the nature of the conversion product.

II. SCALAR THERMAL FIELD AND COVARIANT ACTION

A. Field profile from the Unruh effect

The proper acceleration of a static observer at radius r in Schwarzschild geometry is $a(r) = (GM/r^2)(1 - r_s/r)^{-1/2}$. In the near-horizon limit $r = r_s + \varepsilon$, $\varepsilon \ll r_s$, the Unruh temperature $T_U = \hbar a/(2\pi c k_B)$ reduces to

$$T_U(r) \approx \frac{A}{\sqrt{r - r_s}}, \qquad A = \frac{\hbar c^3}{8\pi\, GM\, k_B \sqrt{r_s}}, \tag{1}$$

where A is derived from the Unruh formula with no free parameters. We promote T_U to a dynamical scalar field $T = T(r)$ with this near-horizon profile.

B. Covariant action

The complete action is

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\, R + \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu T\partial_\nu T - V(T)\right] - \kappa\int d^4x\sqrt{-g}\, T\, \mathcal{T}^\mu_\mu + S_{\rm matter}, \tag{2}$$

where $V(T)$ is derived in Sec. III, κ is fixed in Sec. IV B, and $\mathcal{T}^\mu_\mu$ is the trace of the matter energy–momentum tensor. The coupling $-\kappa T\mathcal{T}^\mu_\mu$ vanishes for pure radiation ($\mathcal{T}^\mu_\mu = 0$ for

a radiation fluid), making the interaction selective to massive matter. This is structurally similar to scalar-tensor theories of the Bergmann-Wagoner class [20, 21], but with a scalar field derived from the thermal structure of the geometry rather than postulated.

Varying Eq. (2) gives the field equations:

$$\Box T = \frac{dV}{dT} + \kappa\, \mathcal{T}^{\mu}_{\mu}, \tag{3}$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\left[\mathcal{T}^{(\mathrm{matter})}_{\mu\nu} + \mathcal{T}^{(T)}_{\mu\nu}\right]. \tag{4}$$

Energy conservation $\nabla_\mu \mathcal{T}^{\mu\nu}_{(\mathrm{total})} = 0$ holds by the Bianchi identity applied to Eq. (4).

III. DERIVATION OF THE EFFECTIVE POTENTIAL

A. Self-consistency condition

For a static spherically symmetric field in Schwarzschild, the d'Alembertian is

$$\Box T = \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(1 - \frac{r_s}{r}\right)\frac{dT}{dr}\right]. \tag{5}$$

With $T = A(r - r_s)^{-1/2}$ and $dT/dr = -\frac{A}{2}(r - r_s)^{-3/2}$:

$$\Box T = -\frac{A}{2r^2}(r - r_s)^{-3/2}\left(\frac{r}{2} - r_s\right). \tag{6}$$

Since $r - r_s = A^2/T^2$, substituting into Eq. (6) yields an expression in T alone:

$$\frac{dV_{\mathrm{eff}}}{dT} = -\frac{T^5(A^2 - r_s T^2)}{4A^2(A^2 + r_s T^2)^2}. \tag{7}$$

The profile $T = A/\sqrt{r - r_s}$ is therefore a self-consistent stationary solution of the Klein–Gordon equation for the potential derived below.

B. Integration

Setting $u = A^2 + r_s T^2$ and integrating Eq. (7):

$$V_{\mathrm{eff}}(T) = \frac{1}{8A^2 r_s^3}\left[\frac{(A^2 + r_s T^2)^2}{2} - 4A^2(A^2 + r_s T^2) + 5A^4 \ln(A^2 + r_s T^2) + \frac{2A^6}{A^2 + r_s T^2}\right]. \tag{8}$$

V_{eff} contains only A and r_s, both determined by M. No free coupling constants enter the geometric sector.

IV. CENTRAL RESULTS

A. Conversion threshold and prediction $r_{\rm conv} = 2r_s$

The extremum condition $dV_{\rm eff}/dT|_{T_c} = 0$ gives $A^2 - r_s T_c^2 = 0$, so

$$T_c = \frac{A}{\sqrt{r_s}}. \tag{9}$$

From $T(r_{\rm conv}) = T_c$:

$$\frac{A}{\sqrt{r_{\rm conv} - r_s}} = \frac{A}{\sqrt{r_s}} \quad \Longrightarrow \quad \boxed{r_{\rm conv} = 2r_s.} \tag{10}$$

This result is entirely determined by M through $r_s = 2GM/c^2$.

The full effective potential including the matter coupling is $V_{\rm eff}(T;\rho) = V_{\rm geom}(T) + \kappa\rho c^2 T$, where $\mathcal{T}^{\mu}_{\mu} \approx -\rho c^2$ for non-relativistic matter. The matter term deforms the potential analogously to Coleman–Weinberg symmetry breaking [12]: the phase transition is not imposed but emerges dynamically from the environmental matter density ρ. In high-accretion regimes, the source term shifts $r_{\rm conv}$ beyond $2r_s$.

Remark 1. *The radius $r_{\rm conv} = 2r_s$ coincides with the innermost bound circular orbit (IBCO) of the Schwarzschild geometry [23]. Whether this coincidence is physically significant is discussed in Sec. XIII.*

B. Coupling constant κ from Hawking consistency

The Hawking energy flux at infinity is [6, 24]

$$\Phi_H = \frac{\hbar c^4}{15360\pi G^2 M^2}. \tag{11}$$

We impose that the outgoing energy flux of the thermal field at $r = 2r_s$ reproduces Eq. (11). The flux balance condition gives

$$\boxed{\kappa = \frac{8\pi G^5 M^5 k_B^3}{15\,\hbar^2 c^{13}}.} \tag{12}$$

The model is now fully determined by M.

C. First-order phase transition

Proposition 1. *The phase transition at $r = 2r_s$ is first-order.*

Proof. Setting $u = r_s T^2/A^2$ and expanding $g(u) = u^{5/2}(1-u)/(1+u)^2$ around $u = 1$:

$$g(1+\epsilon) = -\tfrac{1}{4}\epsilon - \tfrac{3}{8}\epsilon^2 - \tfrac{1}{32}\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{13}$$

Converting to T-derivatives at $T = T_c$:

$$V'''_{\text{eff}}(T_c) = \frac{3A}{4\sqrt{r_s}} \neq 0. \tag{14}$$

Since $V'''_{\text{eff}}(T_c) \neq 0$, the transition is first-order. □ □

A first-order transition implies a latent heat — a discrete release of energy at the conversion surface powering the outgoing flux, consistent with the Hawking consistency condition (12).

V. CONSISTENCY CHECKS

A. Energy conditions at $r = 2r_s$

For a static scalar field: $\rho_T = \frac{1}{2}(T')^2 + V$, $p_r = \frac{1}{2}(T')^2 - V$, $p_\perp = -\frac{1}{2}(T')^2 - V$. With $T' \sim (r - r_s)^{-3/2}$, the kinetic term dominates.

NEC: $\rho_T + p_r = (T')^2 > 0$. ✓

WEC: $\rho_T > 0$. ✓

SEC: reduces to $V_{\text{eff}}(T_c) \leq 0$. At $T = T_c$ with $u = 2A^2$:

$$V_{\text{eff}}(T_c) = \frac{5A^2}{8r_s^3}[\ln(2A^2) - 1] < 0 \tag{15}$$

for astrophysical black holes ($2A^2 \ll e$ in natural units). ✓

All three energy conditions are satisfied. No exotic matter [25] is required.

B. Backreaction

The first-order metric correction at $r = 2r_s$ scales as

$$\delta g_{\mu\nu}\big|_{2r_s} \sim \frac{\ell_P^2}{r_s^2} \ll 1. \tag{16}$$

For a solar-mass black hole this is $\sim 10^{-76}$. The Schwarzschild background remains a valid approximation at the conversion surface.

VI. QUANTUM FLUCTUATIONS AND EMERGENT PLANCK SCALE

TCG is a classical effective theory. Its domain of validity ends where quantum fluctuations of the thermal field become significant.

A. Curvature of the potential at the transition

The second derivative of $V_{\rm eff}$ at $T = T_c$:

$$V''_{\rm eff}(T_c) \approx \frac{3A}{4\sqrt{r_s}}. \tag{17}$$

B. Amplitude of fluctuations: independent of M

The amplitude of thermal fluctuations around T_c:

$$\delta T \sim \sqrt{\frac{k_B T_c}{V''_{\rm eff}(T_c)}} = \sqrt{\frac{4k_B}{3}}. \tag{18}$$

The amplitude is independent of M: all astrophysical black holes produce fluctuations of the same amplitude at their conversion surface.

C. Correlation length: the Planck scale emerges

The correlation length of the fluctuations:

$$\xi_c \sim \frac{1}{\sqrt{V''_{\rm eff}(T_c)}} = \sqrt{\frac{4\sqrt{r_s}}{3A}}. \tag{19}$$

Substituting A from Eq. (1) and evaluating at the boundary $\xi_c = \Delta r \sim \ell_P^2/r_s$:

$$\boxed{\xi_c\big|_{\xi_c=\Delta r} \sim \ell_P.} \tag{20}$$

The Planck length emerges from TCG without being put in by hand. TCG defines its own ultraviolet cutoff.

VII. PHYSICAL SEQUENCE AND SINGULARITY RESOLUTION

The conversion chain is:

$$\text{infalling matter} \xrightarrow{T\nearrow} \text{spaghettification} \xrightarrow{T=T_c} \text{first-order conversion at } 2r_s \xrightarrow{\text{outward}} \text{Hawking radiation.} \tag{21}$$

No matter reaches r_s or $r = 0$.

Conjecture 1 (No Physical Singularity)**.** *In the model with action* (2)*, the Schwarzschild singularity at $r = 0$ is replaced by the first-order conversion surface at $r = 2r_s$. No physical quantity diverges at this surface.*

The Penrose–Hawking singularity theorems [9, 10] assume the strong energy condition. The SEC is satisfied at $r = 2r_s$ (Sec. V.A). The resolution operates by converting matter before it reaches the region where the theorems apply.

VIII. PLANCK REMNANT FROM HAWKING EVAPORATION

As a black hole evaporates, M decreases and $r_{\rm conv} = 4GM/c^2$ contracts. The Hawking evaporation rate [6, 24] is

$$\frac{dM}{dt} = -\frac{\hbar c^4}{15360\pi G^2 M^2}. \tag{22}$$

Setting $r_{\rm conv}(M_*) = \ell_P$:

$$\frac{4GM_*}{c^2} = \ell_P \quad \Longrightarrow \quad \boxed{M_* = \frac{M_P}{4}}. \tag{23}$$

When $M = M_P/4$, the conversion surface reaches exactly ℓ_P — the ultraviolet cutoff of TCG. Below this mass, TCG makes no prediction. The existence of Planck-scale remnants is a generic prediction of many quantum gravity approaches [26, 27]. TCG predicts the same conclusion with a precise factor of 1/4.

IX. KERR EXTENSION

The Kerr metric in Boyer–Lindquist coordinates has $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = (r - r_+)(r - r_-)$, $r_\pm = r_s/2 \pm \sqrt{r_s^2/4 - a^2}$.

In the equatorial plane, the d'Alembertian gives

$$r_{\rm conv}^{\rm eq} = 2r_+ - r_- = r_+ + \sqrt{r_s^2 - 4a^2}. \tag{24}$$

For $a = 0$: $r_{\rm conv}^{\rm eq} = 2r_s$. ✓

The full angular dependence to first order in a^2:

$$r_{\rm conv}(\theta) = r_+ + \sqrt{r_s^2 - 4a^2} + \frac{2a^2 r_+ \sin^2\theta}{r_s^2} + \mathcal{O}(a^4). \tag{25}$$

The conversion surface is an oblate ellipsoid with polar–equatorial asymmetry:

$$\delta r \equiv r_{\rm conv}(\pi/2) - r_{\rm conv}(0) = \frac{2a^2 r_+}{r_s^2}. \tag{26}$$

This asymmetry is testable with the Event Horizon Telescope [32] and its successors.

X. OBSERVATIONAL PREDICTIONS

Schwarzschild: depletion of matter emission in $r_s < r < 2r_s$, producing a dimmer inner brightness profile compared to standard accretion [30]. The photon ring [31] is unaffected ($\mathcal{T}^\mu_\mu = 0$ for radiation).

Kerr: oblate conversion surface with polar–equatorial asymmetry $\delta r = 2a^2 r_+/r_s^2$. The conversion radius lies at the inner edge of the zone contributing to EHT brightness profiles, consistent with current data while testable with ngEHT [33].

High-accretion: the matter source term shifts $r_{\rm conv}$ beyond $2r_s$, producing a larger depletion zone — the most immediate observational test.

XI. RECOVERY OF GENERAL RELATIVITY

For $r \gg r_s$: $T \approx 0$, $\nabla T \approx 0$, $\mathcal{T}^{(T)}_{\mu\nu} \approx 0$. The action reduces to Einstein–Hilbert and all classical tests of GR — perihelion precession, light deflection, gravitational wave observations [34] — are unaffected.

XII. OPEN PROBLEMS

1. **Nature of the conversion product**: What matter converts into at $r_{\rm conv}$ is left open. One candidate is the Penetron [1].

2. **Deeper origin of κ**: Fixed here by external Hawking consistency. A derivation from the action's first principles remains open.

3. **Full metric solution**: Backreaction beyond first-order perturbation theory has not been computed.

4. **IBCO coincidence**: The coincidence $r_{\rm conv} = 2r_s = \text{IBCO}$ and its Kerr analog deserve systematic investigation.

5. **High-spin Kerr**: Equation (25) is first-order in a^2. A full numerical solution is required for $a \approx r_s/2$.

6. **Origin of $V_{\rm eff}$**: The potential is reverse-engineered from self-consistency. Whether a more fundamental action yields it directly is open.

XIII. DISCUSSION

Starting from the Unruh effect alone, TCG constructs a scalar-tensor theory in which every parameter is determined by M:

Quantity	Source
A	Unruh formula, Eq. (1)
$V_{\rm eff}$	Self-consistency, Eq. (8)
T_c, $r_{\rm conv} = 2r_s$	Stationarity, Eq. (10)
κ	Hawking consistency, Eq. (12)
Transition order	$V'''_{\rm eff}(T_c) \neq 0$, Eq. (14)
$\delta T \sim \sqrt{k_B}$	Universal, Eq. (18)
$\xi_c \sim \ell_P$	Emergent cutoff, Eq. (20)
$M_* = M_P/4$	Planck remnant, Eq. (23)
$r^{\rm Kerr}_{\rm conv}(\theta)$	Kerr extension, Eq. (25)

TABLE I. All TCG predictions, fully determined by M.

TCG differs from existing regular black hole models [13–15] in that the singularity-resolving mechanism is not postulated but derived from a self-consistency condition. It differs from loop quantum black holes [16, 17] in that it operates at the semi-classical level, without quantizing the geometry. It is closest in spirit to black hole models based on non-linear electrodynamics [29] or $f(R)$ gravity [28] in that a modified effective action produces

a regular geometry, but the modification here arises uniquely from the thermal structure of the horizon.

The coincidence $r_{\rm conv} = 2r_s$ = IBCO suggests a possible connection between thermal stability of orbits and the phase transition. In Kerr, the IBCO acquires angular dependence [23]. Whether the conversion surface tracks the IBCO across spin parameters is a concrete and calculable question left for future work.

General relativity remains valid and is recovered exactly for $r \gg r_s$. TCG modifies physics only in the near-horizon regime $r_s < r \leq 2r_s$, precisely where quantum gravity corrections are expected and no direct test has been performed.

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DATA AVAILABILITY

No observational data were used in this work. All results are derived analytically. The LaTeX source and compiled PDF are available at Zenodo [2].

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